Stochastic Optimization Assignment Report 2024/25

Scenario Reduction

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1 Introduction

This project introduces two heuristic methods for clustering similar scenarios, implemented within the ScenarioTree class. The first method is a scenario aggregation heuristic, while the second leverages the Wasserstein distance to measure similarity.These heuristics are tested on two-stage stochastic optimization problems, specifically the newsvendor problem and the two-item assembly-to-order problem.

2 Metodology

Both the newsvendor and assembly-to-order problems are formulated as two-stage linear stochastic programming models that account for demand uncertainty. To solve these problems efficiently, we employ a Sample Average Approximation (SAA)-based approach, which generates a large scenario tree to approximate the underlying probability distribution. The resulting optimization model is solved using the Gurobi solver, providing an optimal solution under the given scenario set. However, due to the computational complexity associated with a large number of scenarios, we apply heuristic scenario reduction methods to obtain a more tractable problem while preserving solution quality. The effectiveness of the scenario reduction process is evaluated through in-sample and out-of-sample stability analysis, ensuring the robustness of the reduced scenario tree. Specifically, starting from two different scenario reduction approaches, the in-sample and out-of-sample stability are computed iteratively. The final reduced scenario tree size is determined as the minimum dimension that satisfies both stability criteria, ensuring a balance between computational efficiency and solution accuracy. If the algorithm fails to find out-sample stability the dimension is set equal to the in sample stability dimension. If it fails to find both stability is set equal to the default value of 50 (1 % of the minimun dimension on which the heuristics are tested)

3 Model

3.1 Demand

Each item is assumed to follow an independent binomial distribution, expressed as:

$$D_j(\omega) \sim Binom(n_j, p_j) \forall j \in J$$

and

$$D_j \perp\!\!\!\perp D_i \forall j \neq i$$

where n_j represents the maximum demand of item j and p_j is the probability of selling each unit. Consequently, the stochastic model for scenario tree generation follows a general binomial framework, where observations are drawn from a binomial distribution, and each scenario occurs with equal probability:

$$p_s = \frac{1}{s} \forall s \in \mathcal{S}$$

3.2 Aggregation Heuristics

This heuristic aggregates scenarios by grouping them based on observed demand values. Specifically, n bins are created by equally spacing the demand values between the minimum and maximum observed demand. Each scenario is then assigned to a bin based on its demand level, effectively clustering similar demand realizations to reduce the size of the scenario tree. The demand value associated with each bin is computed as the average demand of all observations within that bin. The probability of each bin is determined as the sum of the probabilities of all observed scenarios assigned to it. This approach ensures that the aggregated scenarios retain the overall distribution characteristics while significantly reducing the complexity of the scenario tree. When handling multi-item demand values, the binning edges are determined based on the total demand, which is computed as the sum of the observed demand values across all items. This ensures that the binning process captures overall demand variations while maintaining a structured and consistent scenario aggregation approach.

3.3 Wasserstein Distance

The Wasserstein-based heuristic reduces the number of scenarios by selecting a subset that closely approximates the original distribution while minimizing the Wasserstein distance. This heuristic effectively finds the optimal transport of probability mass from the original set of scenarios to the reduced subset. In theory, the exact solution would require evaluating all possible subsets of a given size and computing the Wasserstein distance for each one, selecting the subset that minimizes the distance. However, this approach has a combinatorial complexity of: $O\left(\binom{n}{s}\right)$ which becomes computationally infeasible for large scenario trees. To address this challenge, an approximate approach has been adopted. Instead of evaluating all subsets, a large but finite set of subsets is randomly sampled, and the Wasserstein distance is computed only for this sampled subset. This significantly reduces computational time. When handling multi-item demand values, the points in the source distribution are computed as the sum of the different observations, which may impact the preservation of individual demand interactions.

4 Numerical Experiments

4.1 Newsvendor Problem

```
# Define binomial demand model
total_demand = [1000000] # Total possible demand for every item
success_prob = [0.1] # chance of each unit being bought
cost = 1
selling_price = 10
seed1 = 301286
seed2 = 302642
news_vendor = GeneralBinomialModel(total_demand, success_prob,
    seed1)
```

Those are the parameters selected for testing the scenario reduction on the newsvendor problem.

4.1.1 Results

Method	Scenarios	Stability	Achieved in	toll
Aggregation	500	In sample	2 scenarios	1e-3
		Out sample	24 scenarios	1e-6
		Both	24 scenarios	
	1000	In sample	24 scenarios	1e-3
		Out sample	25 scenarios	1e-6
		Both	25 scenarios	
	10000	In sample	2 scenarios	1e-3
		Out sample	16 scenarios	1e-6
		Both	16 scenarios	
	100000	In sample	2 scenarios	1e-3
		Out sample	48 scenarios	1e-6
		Both	48 scenarios	
Wasserstein	100	In sample	2 scenarios	1e-3
		Out sample	2 scenarios	1e-3
		Both	2 scenarios	
	500	In sample	24 scenarios	1e-3
		Out sample	25 scenarios	1e-3
		Both	25 scenarios	
	1000	In sample	2 scenarios	1e-3
		Out sample	16 scenarios	1e-3
		Both	16 scenarios	

Table 1: Stability analysis

Method	Initial scenarios	Reduced	In. obj	Fin. obj.	In. opt.	Fin. opt
Aggregation	500	24	899452.02	899452.02	100359	100391
	1000	25	899399.44	899399.90	100361	100397
	10000	16	899428.33	899428.03	100378	100303
	100000	48	899462.47	899462.70	100383	100359
Wasserstein	100	2	899119.90	899119.90	100359	100310
	500	25	899452.02	899119.90	100359	100310
	1000	16	899399.44	899119.90	100361	100310

Table 2: Scenario reduction

4.2 Assembly-to-Order Problem

```
# Define binomial demand model
total_demand = [1000, 5000] # Total possible demand for every item
success_prob = [0.1, 0.05] # chance of each unit being bought
# Given parameters
costs = [1, 1, 3]  # Component costs
selling_prices = [6, 8.5]  # Selling prices of final products
G = [[1, 1, 0],  # Item 0 needs components
     [1, 1, 1]]  # Item 1 needs components
work_time = [[0.5, 0.25, 0.25]] # Time required to produce each
#component per machine
time_a = [500]  # Time availability
seed1 = 301286
seed2 = 302642
```

Those are the parameters selected for testing the scenario reduction on the two items ATO problem.

4.2.1 Results

Method	Scenarios	Stability	Achieved in	toll
Aggregation	500	In sample	50 scenarios	1e-3
		Out sample	not reached	1e-3
		Both		
	1000	In sample	not reached	1e-3
		Out sample	not reached	1e-3
		Both		
	10000	In sample	not reached	1e-3
		Out sample	not reached	1e-3
		Both		
	100000	In sample	not reached	1e-3
		Out sample	not reached	1e-3
		Both		
Wasserstein	100	In sample	10 scenarios	1e-3
		Out sample	not reached	1e-3
		Both		
	500	In sample	5 scenarios	1e-3
		Out sample	not reached	1e-3
		Both		
	1000	In sample	not reaached	1e-3
		Out sample	not reached	1e-3
		Both		

Table 3: Stability analysis

Method	Initial scenarios	Reduced	In. opt.	Fin. opt. order
Aggregation	500	50	(392, 392, 284)	(392, 392, 284)
	1000	50	(396, 396, 283)	(394, 394, 276)
	10000	50	(403, 403, 291)	(402, 402, 280)
	100000	50	(407, 407, 295)	(408, 408, 286)
Wasserstein	100	10	(388, 388, 272)	(376, 376, 260)
	500	5	(392, 392, 284)	(371, 370, 270)
	1000	50	(396, 396, 283)	failed to converge

Table 4: Scenario reduction

5 Conclusions

The aggregation method performs significantly faster in large-scale problems, while the Wasserstein-based method struggles due to the increasing computational cost of evaluating subsets and computing the cost matrix for Wasserstein optimization.

The heuristic aggregation approach consistently outperforms Wasserstein clustering in the newsvendor problem, achieving high stability across different scales. However, in the assembly-to-order problem, it performs less effectively in terms of stability, primarily due to the strong dependence of the objective function on constraints, which causes different observations to interact. In the multi-item aggregation heuristic, summing over observations eliminates individual scenario details, thereby losing information about their interactions within the constraints.

Due to the stability issues observed in the assembly-to-order problem, the quality of scenario reduction is evaluated based on the closeness between the first-stage decision variable of both the full and the reduced scenario models.

In the assembly-to-order optimization, the Wasserstein heuristic performs slightly better in terms of stability, but the aggregation method still outperforms Wasserstein clustering in terms of closeness to the initial optimal first-stage decision variable§